American Journal of Applied Science and Technology

VOLUME Vol.05 Issue 07 2025
PAGE NO. 73-75
DOI 10.37547/ajast/Volume05Issue07-12

Ant'ye, Mantissa and Its Applications (Continued)

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Received: 31 May 2025; **Accepted:** 29 June 2025; **Published:** 31 July 2025

Abstract: This article provides theoretical and practical information as well as proofs about the antyle and mantissa, and highlights the main points of it. It also suggests methods for solving several complex and Olympic-type problems with the help of the antyle and mantissa.

Keywords: Ant’e, mantissa, sequences, functions in R set, definition of integral, canonical decomposition, bounded functions.

Introduction:

Ant'e(French. Entier) The term integer was proposed by Legendre in 1798 and has been in use for two hundred years. The new concept came to his mind from the need to count how many times the prime number p occurs in the canonical distribution of a complex number n.

The designation of the ant’e x as [x] was proposed by Gauss in 1808.

There is almost no significant historical information in the literature about the term mantissa.

In the old days, ant'e and the mantissa Problems on (whole and fractional parts of a number) were considered very narrow Olympiad problems. Recently, while easy problems of this type have been offered in entrance exams to universities in some foreign countries, problems with a higher level of difficulty have begun to be offered to students in national and international Olympiads.

So what's so interesting about the ant’e and mantissa issues?

If we consider the ant’e and mantissa as functions, these functions are not continuous, but rather, piecewise continuous. The existence of such functions is fundamentally different from conventional solving methods.

The uniqueness of the topic "Ant’e and Mantissa" and the complexity, or rather, the difficulty of the problems on this topic, is that the student does not have enough practice in this area. The solution to most of these types of problems is based on a very short logical analysis.

In addition, the ant’e and mantissa have properties that require the student to perform a number of operations. Despite these characteristics, many of the properties can be derived while solving problems.

The article presents selected problems on ant’e and mantissa typical of the number theory and algebra section of the subject, and also includes some problems recommended in national Olympiads in foreign countries. For those interested, the article provides complete and substantiated solutions to the problems.

1. $[\boldsymbol{f}(\boldsymbol{x})] = \boldsymbol{g}(\boldsymbol{x})$ Methods for solving equations in the form of.

The equations in question are often found in competitive exams. Equations of this form are also given other equations, for example, equations involving the mantissa.

$$[\boldsymbol{f}(\boldsymbol{x})] = \boldsymbol{g}(\boldsymbol{x}) \qquad (1.1)$$

The uniqueness of equations of the form is that the right-hand side of the equality is an integer. From this it follows that the equation is satisfied only by values of the variable that take on integer values of the function. In addition, if the value domains of the functions are finite, equation (1.1) can be decomposed into several simpler equations.g$(x)f(x)$ $va\ g(x)$

1.1. a method of applying a function that is the inverse of a function.$g(x)$

Since equation (1.1) is satisfied by the values of the variable that make the function complete,g(x)

$$n = g(x)\,, \qquad n \in Z$$

Let us introduce the definition. Let us assume that is a strictly monotonic (increasing) function. Then the function is invertible and we can express it by:$g(x)g(x)x\ n$

$$x = g^{-1}(n).$$

As a result, by defining the variable twice, equation (1.1) is reduced to an equation with all variables.

$$[f(g^{-1}(n))] = n, \quad \left[f\left(g^{-1}(n)\right) - n\right] = 0 \qquad (1.2)$$

The last equation is equivalent to the following double inequality

$$0 \le f\left(g^{-1}(n)\right) - n < 1\,, \qquad (1.3)$$

We can solve it by first finding the number of solutions of (1.1) and then finding the roots of the given equation by inverse substitution.n

The scope of application of the method considered above to equations of the form (1.1) is somewhat limited by the invertibility of the function and the complexity of the inverse substitution of variables compared to other methods.$g(x)$

Now let's apply this method to some problems of the form (1.1)

11. Solve the equation. $\left[\frac{x+1994}{81}\right] = \frac{x+2011}{101}$

Solution. We can introduce the notation . Then$n = \frac{x+2011}{101} x = 101n - 2011$

Now we can write the double inequality

$$0 \le \frac{101n - 17}{81} - n < 1.$$

By solving the left side of the double inequality (which only accepts integer values), we obtain, and by solving the right side, we obtain. This means that the equations have four solutions.$n \ge 1 n\ n \le 4 n = 1{,}2{,}3{,}4$

Answer: -1910, -1809, -1708, -1607

12. Solve the equation.$[x] = \frac{\sqrt{2}}{x}$

Solution. We can introduce the notation . Then $n = \frac{\sqrt{2}}{x} x\ x = \frac{\sqrt{2}}{n} n$

We solve the double inequality with respect to the auxiliary integer by expressing it through n

$$n \le \frac{\sqrt{2}}{n} < n + 1.$$

First, let's consider the case where the values are positive. In this case, the inequality becomes$n$

$$n^2 \le \sqrt{2} < n^2 + n.$$

It seems obvious that this could be the whole solution. $n = 1$

Now the inequality in the negative case is this n

$$n^2 + n < \sqrt{2} \le n^2$$

It turns out that this inequality, in turn, has no solution in s: if the "right" part of the inequality in s does not hold, then the inequality on the left side of the inequality in s does not hold. $n \epsilon Z < 0\ n = -1\ n \le -2$

So, the equation given in has a unique solution. $n = 1$

Answer: $\sqrt{2}$

13. How many solutions does the equation have? $x = \{32x\}$

Instructions: Write the equation given initially in the form.$[32x] = 31x$

After the definition, you can write an inequality. This means that the given equation has 31 solutions.$n = 31x 0 \le n < 31$

Answer: 31 solutions.

1.2. Equipotent substitution method.$\boldsymbol{0 \le f(x) - g(x) < 1}$

$f(x)$ The main property of the mantissa of a function is this

$$0 \le \{f(x)\} < 1, \quad \text{yoki} \ \ 0 \le f(x) - [f(x)] < 1$$

consists of relationships.

Now we can write these equally strong substitutions for the equation $[f(x)] = g(x)$

$$[f(x)] = g(x) \quad \Leftrightarrow \quad \begin{cases} 0 \le f(x) - g(x) < 1 \\ \quad g(x)\ \epsilon\ Z \end{cases} \qquad (1.4)$$

The solution of inequality (1.4) can consist of one or more intervals, and in this case the solutions of equation (1.1) belong to the set of solutions of inequality (1.4). Therefore, from the above considerations it follows that inequality (1.4) is a consequence of equation (1.1). It also follows that if inequality (1.4) has no solution, equation (1.1) also has no solution.

Now we will solve problems using the equivalent substitution method.

14. (All-Russian./2010-2011) Solve the equation$x^2 - [x] - 2 = 0.$

Solution. The given equation consists of an equation of the form. We can reduce equation (1`.4) to the form$[f(x)] = g(x)$

$$0 \leq x - x^2 + 2 < 1.$$

The solution to the inequality on the left is a segment (we will not consider the solutions that arise from the inequality on the right, since it is a "not very convenient discriminant"). Substituting the four values obtained from this into the given equation, we obtain the following.$[-1,2]g[x] \epsilon \{-1,0,1,2\}$.

$$x_1 = -1 \quad ([x] = -1 \text{ bo'lganda}), x_2 = \sqrt{3} \; ([x] = 1 \text{ bo'lganda}),$$
$$x_3 = 2 \; ([x] = 2 \text{ bo'lganda}),,$$

Answer: $\{-1, \sqrt{3}, 2\}$.

15. Solve the equation.$[2x] = x^2$

Solution. As a result of an equally strong substitution, this

$0 \leq 2x - x^2 < 1,$

we form the inequality. The solution to this inequality is$x \epsilon [0,1) \cup (1,2]$.

Now it is not difficult to choose from the generated half-intervals those that make the value of integer.x^2

Answer: .$\{0, \sqrt{2}, \sqrt{3}, 2\}$

1.3. Method of bounded functions.$\boldsymbol{f(x)}$ **va** $\boldsymbol{g(x)}$

If the functions are bounded, the right (left) sides of equation (1.1) take on a finite number of integer values. In this case$f(x)$ va $g(x)$

Equation (1. 1) is reduced to the set of these systems of equations

$$\begin{cases}[f(x)] = n_1, \\ g(x) = n_1,\end{cases} \quad \begin{cases}[f(x)] = n_2, \\ g(x) = n_2,\end{cases} \quad \dots \quad \begin{cases}[f(x)] = n_k, \\ g(x) = n_k,\end{cases} \quad (1.5)$$

In this case, the range of values of the function is either the integers in the range of values or this equality consists of$n_1, n_2, \dots, n_k$ lar $[f(x)]g(x)$ funksiyaning

$$\{n_1, n_2, \dots, n_k\} = E([f(x)]) \cap E(g(x)) \cap Z \qquad (1.6)$$

In other words, boundedness means that one function is bounded from above and the other from below. The analysis of this case also leads to the set (1.5).$f(x)$ va $g(x)$

It is worth noting that the set (1.5) is an equally strong permutation, only (1.6) is satisfied. However, it is not always practical to construct the set. In this case, the set (1.5) consists of the result of equation (3.1).$\{n_1, n_2, \dots, n_k\}$

16. Solve the equation $[log_2 x] = cosx$.

Solution. We use the fact that the function is bounded and takes integer values of . We write down the set (1.5) $cosx - 1, \; 0$ va 1

$$\begin{cases}[log_2 x] = -1, \\ cosx = -1,\end{cases} \quad \begin{cases}[log_2 x] = 0, \\ cosx = 0,\end{cases} \quad \dots \quad \begin{cases}[log_2 x] = 1, \\ cosx = 1.\end{cases}$$

Let's look at the second system. This system gives the only solution to the given equation. The solution to the equation consists of a half-interval, and the solution to the equation is in this, that is, we paid attention to the domain of definition of the logarithmic function. We can verify that $[log_2 x] = 0 \; [1, 2) cosx = 0 \;\; x = \frac{\pi}{2} + 2\pi n, n \epsilon Z, x > 0 \left[log_2 \frac{\pi}{2}\right] = 0$

The remaining two systems have no solution.

Answer: $\left\{\frac{\pi}{2}\right\}$

2. Examples for independent work.

2.1. Solve the equation $x^2 - 3x = [3 - 2x]$

2.2. Solve the equation $x^2 = [2x]$

2.3. Solve the equation $\left[3\sqrt{x - x^2}\right] = 2^{x - \frac{1}{2}}$

2.4. Solve the equation $[x^6 - 1] = -\frac{9(x-1)^2}{4}$

2.5. Solve the equation $[3x] = log_{0,5} x$

2.6. Solve the equation $[2 - x] = 2sinx$

2.7. Solve the equation $[2x] = x^3 - 3$.

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