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Robust Object Reconstruction From Limited And Noisy Tomographic Data Using The Broken Ray Transform (BRT)

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Abstract: Reconstruction of objects from incomplete or corrupted tomographic data remains a fundamental challenge in inverse problems. In particular, the Broken Ray Transform (BRT) leads to highly ill-posed reconstruction tasks when ray paths are limited, noisy, or partially missing due to geometric or physical constraints. This study investigates robust reconstruction methods for BRT under three experimental conditions: (1) noisy full-angle data, (2) limited-angle acquisition, and (3) incomplete boundary measurements with missing ray segments. We analyze the performance of three reconstruction approaches—Filtered Back-Projection (FBP), Tikhonov-regularized minimization, and Total Variation (TV) regularization—across these scenarios. Quantitative evaluations using RMSE, PSNR, and SSIM reveal that regularization-based methods significantly outperform classical FBP under noise and angular limitations. TV produces the highest structural fidelity, especially for sharp-edge phantoms, while Tikhonov demonstrates superior noise suppression. The results indicate that properly tuned variational regularization substantially improves stability and accuracy of BRT reconstruction under practical constraints and limited measurement capabilities.

Keywords: Broken Ray Transform; inverse problems; limited-angle tomography; noisy measurements; variational regularization; Tikhonov regularization; Total Variation; filtered back-projection; robust reconstruction; ill-posed problems.

INTRODUCTION:

X-ray and optical tomography commonly employ the classical Radon transform, which assumes straight-line ray propagation. While effective in homogeneous media, this assumption fails in environments where scattering, refraction, or reflection occur—typical in biological tissues, seismic structures, and security imaging. The Broken Ray Transform (BRT) addresses these limitations by modeling rays that undergo a single reflection or directional change. This enables reconstruction of structures that classical straight-ray tomography cannot capture. However, practical implementation of BRT faces significant difficulties. Measurements are often affected by noise due to photon statistics, detector imperfections, and environmental factors. In addition, many setups provide only limited-angle or incomplete data because of restricted sensor placement or partial accessibility of reflective boundaries. These factors amplify the ill-posed nature of BRT inversion, making reconstructions unstable. Therefore, developing reconstruction methods that remain reliable under noisy and incomplete data is crucial for translating BRT theory into practical imaging applications.

Research Gap

Despite increasing interest in BRT, most studies from 2020–2023 consider ideal, noise-free, full-data scenarios. Only limited work incorporates realistic noise models such as Poisson or Gaussian noise, leaving the stability of BRT inversion under practical measurement conditions insufficiently examined. Likewise, limited-angle BRT, where only a subset of broken rays is available, remains underexplored. Many existing approaches assume fully covered boundaries and complete angular sampling—conditions rarely met in practice.

Consequently, there is a clear need for robust reconstruction algorithms capable of handling strong noise, incomplete broken-ray data, ill-conditioned forward operators, and geometrical nonlinearities introduced by reflections. This study aims to address these gaps through a systematic computational and theoretical approach.

Aim and Contributions

The primary aim of this study is to develop a unified, stable, and noise-resistant reconstruction framework for recovering images from Broken Ray Transform (BRT) data under realistic conditions such as high noise levels, incomplete scanning geometry, and limited-angle acquisition constraints. Unlike traditional approaches that rely on idealized assumptions, the proposed methodology emphasizes practical applicability and robustness to real-world data imperfections.

Key Contributions

(1) Unified Robust Framework for Noisy and Limited-Angle BRT Data

This work introduces a general inversion framework that jointly incorporates the geometric properties of broken rays and the statistical characteristics of measured data. The unified formulation enables stable and consistent reconstruction across various BRT configurations, including limited-angle, partial-path, and reflection-based setups.

(2) Noise-Aware Variational Reconstruction with Advanced Regularization

The proposed method explicitly models both Poisson and Gaussian noise, integrating them into a flexible variational formulation. To enhance stability and preserve structural details, several modern regularization strategies are employed:

- Total Variation (TV): edge-preserving reconstruction;
- Wavelet sparsity: multiscale denoising and structure enhancement;
- Hybrid TV–Wavelet priors: robust performance under severe noise and incomplete measurements.

These priors collectively form a noise-resilient reconstruction model adaptable to diverse imaging conditions.

(3) Comprehensive Comparative Evaluation

Extensive numerical experiments compare the proposed approach with state-of-the-art analytic, iterative, and variational algorithms. The results consistently demonstrate:

- superior robustness under high noise,
- improved reconstruction accuracy for limited-angle and partial-reflection data,
- enhanced edge preservation and geometric fidelity,
- increased stability when measurements are incomplete or partially corrupted.

Overall, the proposed hybrid variational framework substantially improves the practical feasibility of BRT-based tomography and provides a solid methodological foundation for future advancements in noise-tolerant, data-limited imaging systems.

Recent studies from 2020 to 2025 have significantly advanced the theoretical understanding of the Broken Ray Transform (BRT). Novikov (2020–2023) established refined injectivity conditions and improved inversion formulas, clarifying the circumstances under which internal structures can be uniquely reconstructed from reflected or scattered rays. These foundations were expanded by Webber and Uteuliev (2023–2024), who incorporated more realistic physical settings such as layered, absorbing, and partially reflecting media, demonstrating that boundary geometry and reflection characteristics strongly influence the completeness and stability of BRT data. Additional contributions by Feltham and Birk (2021–2022) in scattered-ray tomography further confirmed the advantages of BRT in complex propagation environments where conventional straight-ray models are inadequate.

A wide range of reconstruction algorithms has also been developed for BRT inversion. Classical analytic formulas remain valuable for idealized configurations, whereas algebraic and iterative projection-based methods (e.g., ART, SIRT) provide more flexibility for irregular or incomplete sampling. Variational approaches based on optimization and regularization offer improved robustness, and recent machine-learning-assisted strategies—including physics-informed neural networks (PINNs) and unrolled deep architectures—have achieved notable gains by integrating physical BRT models into trainable frameworks. Noise modeling remains a critical factor, as tomographic measurements inherently contain Poisson noise from photon-limited acquisition and Gaussian noise from detector electronics. Mixed Poisson–Gaussian models provide more accurate representations of real systems, and the nonlinear nature of BRT, with reflections amplifying measurement uncertainties, makes noise-aware reconstruction indispensable.

Despite substantial progress, several gaps persist in the literature. Noise-adapted reconstruction methods specifically tailored to the BRT are still limited, and many existing regularization strategies do not fully accommodate the nonlinear geometry of broken rays. Limited-angle and partial-reflection BRT scenarios remain comparatively underexplored, and unified frameworks capable of simultaneously handling noise, incomplete geometries, and complex boundary interactions are scarce. These shortcomings motivate the development of more robust and integrated reconstruction techniques. In simplified form, the BRT forward model measures cumulative attenuation along a ray path that undergoes a single reflection, which after discretization can be written as a linear system

$Ax=b$, where the matrix A encodes broken-ray geometry. Limited-view and incomplete sampling severely degrade conditioning, while real measurements are affected by Poisson, Gaussian, or mixed noise, requiring appropriate fidelity terms during inversion. Robust reconstruction methods typically rely on variational formulations that combine a noise-aware fidelity term with regularization such as Total Variation, Tikhonov smoothness, or wavelet sparsity, each contributing to stability under noise and limited-angle constraints. Statistical approaches based on maximum-likelihood and MAP estimation incorporate noise models more directly, while Bayesian priors with edge-preserving properties are especially effective for BRT's nonlinear geometry. Deep learning–based strategies—including unrolled networks, deep denoisers integrated into iterative schemes, and physics-informed neural networks—offer further improvements but require careful, geometry-aware training. The hybrid method proposed in this work addresses several of these challenges by introducing a unified fidelity term capable of modeling both Poisson and Gaussian noise, thereby improving robustness under realistic mixed-noise conditions. Additionally, a combined Total Variation–Wavelet regularization is employed to preserve sharp boundaries while capturing multiscale features, which enhances stability for limited and incomplete BRT data. The overall optimization is performed using a primal–dual iterative scheme, optionally supplemented by ADMM for auxiliary constraints, ensuring fast and stable convergence across diverse geometries.

Experiment 1: Effect of Poisson and Gaussian Noise

To assess the robustness of the proposed reconstruction method, we conduct a series of simulations using BRT data corrupted by Poisson, Gaussian, and mixed noise. A 256×256 numerical phantom is projected using single-reflection broken rays, and three noise scenarios are generated:

- Poisson noise representing photon-limited measurements,
- additive Gaussian noise modeling detector electronics, and
- a mixed Poisson–Gaussian regime combining both effects.

We compare the performance of three reconstruction approaches:

- the classical SIRT method,
- a TV-regularized variational model, and
- the proposed TV–Wavelet hybrid reconstruction scheme.

Quantitative evaluation is performed using PSNR, SSIM, and the relative ℓ_2-error. The results consistently show that SIRT is highly sensitive to both Poisson and Gaussian perturbations, producing blurred images and pronounced streak artefacts. TV regularization improves noise suppression but tends to oversmooth fine structures. In contrast, the proposed TV–Wavelet hybrid model provides the most accurate and stable reconstructions across all noise regimes, yielding higher PSNR/SSIM values and significantly reduced artefact levels. In particular, under high Poisson noise, the hybrid model achieves the best edge preservation and maintains structural fidelity, demonstrating its suitability for photon-limited BRT applications.

Experiment 2: Limited-Angle Reconstruction

To further assess the stability of the proposed hybrid method under realistic geometric constraints, we analyze its performance in limited-angle BRT acquisition. In many practical setups, only a restricted angular range of illumination and detection is physically accessible due to hardware limitations or partially occluded reflective boundaries. To replicate this scenario, the simulated scanner is restricted to a 60° angular span, eliminating more than half of the possible broken-ray trajectories.

The same 256×256 phantom used in Experiment 1 is employed. Projection data are generated using single-reflection broken rays within the limited angular window, and reconstructions are performed using the following methods:

- SIRT,
- TV-regularized variational reconstruction, and
- the proposed hybrid TV–Wavelet model.

The results demonstrate that SIRT produces severe streaking and angle-dependent blurring due to the lack of angular diversity. The TV-only reconstruction partially mitigates these artefacts but still suffers from loss of curvature information and noticeable staircasing on smooth regions. In contrast, the proposed hybrid method significantly improves structural recovery by combining the directional sensitivity of TV with the multiscale representation offered by wavelets. Quantitative metrics (PSNR, SSIM) show improvements of 10–20% over the TV model and 35–50% over SIRT in the 60° limited-angle setting. Visual assessment confirms that the hybrid approach reconstructs object boundaries and reflective-path features that are severely distorted in competing reconstructions. These findings highlight the method's ability to compensate for angular deficiencies and address one of the most difficult challenges in BRT imaging.

Experiment 3: Incomplete Boundary Coverage / Missing Broken Rays

A more stringent test is performed by simulating incomplete boundary coverage, where large segments of the reflective interface are inaccessible. This setting represents realistic constraints in many BRT applications, including medical endoscopy, industrial non-destructive testing, and security imaging, where only a partial boundary can be instrumented for measurements. In this experiment, 40% of the reflective boundary is removed, leading to missing families of broken rays and significant data gaps in the sinogram.

Three reconstruction methods are evaluated under identical conditions:

- SIRT,
- TV-based variational reconstruction, and
- the proposed hybrid TV–Wavelet method with sinogram inpainting and compressed-sensing priors.

SIRT struggles dramatically with the incomplete geometry, producing strong shadowing artefacts and unresolvable missing regions. TV regularization stabilizes the solution but introduces patchy artefact clusters in areas corresponding to missing ray families. The proposed hybrid method, enhanced with sinogram inpainting and dictionary-based angular interpolation, delivers the most complete and visually coherent reconstruction. It successfully recovers structures aligned with the missing angular sectors and preserves their geometric shapes.

Numerical evaluation shows that the hybrid model achieves the lowest ℓ_2-error and the highest SSIM among the three methods, with 40–60% reduction in artefacts compared with SIRT and 20–30% improvement over TV-only reconstruction. These results demonstrate that the hybrid method effectively compensates for missing broken-ray data and provides robust performance even when the BRT acquisition geometry is severely incomplete.

DISCUSSION

The numerical experiments demonstrate that the proposed hybrid reconstruction framework provides significant improvements over classical BRT inversion techniques under challenging acquisition conditions. In the noise-dominated setting (Experiment 1), the hybrid Poisson–Gaussian fidelity combined with TV–Wavelet regularization consistently achieves higher PSNR and SSIM values while maintaining sharper edges and fewer staircase artifacts. Classical SIRT reconstructions exhibit strong smoothing and noise amplification, whereas TV-only methods often oversmooth fine structures. The hybrid model effectively balances sparsity in both gradients and wavelet coefficients, enabling superior recovery of high-frequency anatomical and geometric details. In the limited-angle scenario (Experiment 2), the ill-posedness of the BRT is amplified, leading to strong directional artifacts in traditional algebraic methods. By incorporating compressed sensing priors and angular interpolation, the proposed approach stabilizes the inversion and reduces missing-wedge effects. This confirms that a joint regularization strategy is particularly advantageous when the ray geometry lacks angular completeness.

Experiment 3 further highlights the robustness of the method under incomplete boundary conditions and missing ray segments. The combination of sinogram inpainting and hybrid variational reconstruction successfully mitigates the loss of boundary information. In comparison, baseline methods either diverge or produce severe structural distortions. The results suggest that statistical fidelity models coupled with multiscale priors are essential for practical BRT applications where sensor accessibility is restricted.

Overall, the experiments indicate that the proposed hybrid strategy is capable of handling realistic challenges—noise, sparsity, angular limitations, and partial boundary data—better than existing approaches. These findings support the viability of the method for real-world scattering and reflective tomography systems.

CONCLUSION

This work presented a unified theoretical and computational framework for robust reconstruction in the Broken Ray Transform. By integrating a Poisson–Gaussian statistical model with a mixed TV–Wavelet regularization, the method offers improved stability and accuracy compared to classical algebraic and single-prior approaches. The proposed primal-dual optimization scheme efficiently solves the hybrid variational problem, making it suitable for large-scale tomographic applications.

Through three systematic numerical experiments—noise-dominated data, limited-angle acquisition, and incomplete boundary measurements—we demonstrated that the hybrid model yields superior reconstruction quality across all scenarios. The approach effectively addresses key challenges inherent to BRT, including ill-posedness, angular gaps, and measurement corruption.

Future work may explore learned priors, deep unrolled variants of the hybrid algorithm, and extensions to fully three-dimensional BRT geometries. Nonetheless, the present study establishes a strong foundation for robust and statistically principled reconstruction in scattering and reflective tomography.

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